Explicit Finite element approximation and invariant domain properties of hyperbolic systems

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Problem & Algorithm

Problem: Compressible Euler Equations

$$\begin{cases} \partial_t \rho + \partial_{x_j}(\rho u_j) & = 0, \\ \partial_t(\rho u_i) + \partial_{x_j}(\rho u_i u_j + p\delta_{ij}) & = 0, \\ \partial_t E + \partial_{x_j}((E+p)u_j) & = 0, \end{cases}$$

where $E = \rho e + \frac{1}{2}\rho u^2$ is the total energy, and e is the special internal energy.

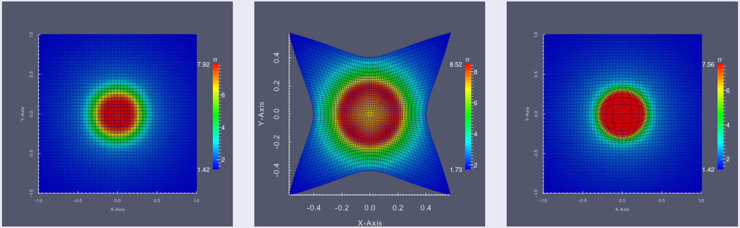
1. Equation of state: ideal gas $p = (\gamma - 1)\rho e$.
2. Proper initial and boundary condition.

Two Algorithms

1. The first method keep the positivity of density and internal energy;
2. The second method depends on local Riemann solver and keeps the invariant domain property;
3. The second method can be generalized to other hyperbolic systems;
4. Both of methods can be applied in Eulerian coord. , Lagrangian coord., and ALE coord with a little modification.

Test & Outlook

2d NOH problem



Future work

- Incorporate entropy viscosity.
- Advanced algorithm to move the mesh.
- Add remap stage.
- Extension of limiters to systems.
- Extension to DG (with no Riemann solver, exact or approximate).
- Convergence analysis, error estimates, BV bound on general meshes.