New Families of H(div) mixed Finite Element on Quadrilaterals and Hexahedra

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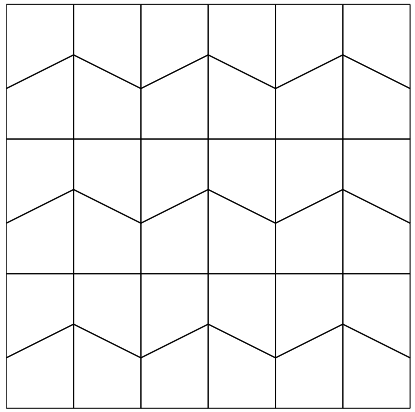
Single phase flow in porous media:

$$\begin{aligned} \mathbf{u} &= -a_\epsilon \nabla p \qquad && \text{in } \Omega \\ \nabla \cdot \mathbf{u} &= f && \text{in } \Omega \\ \mathbf{u} \cdot \nu &= 0 && \text{on } \partial\Omega \end{aligned}$$



Loss of convergence for $\nabla \cdot \mathbf{u}_h$ for RT and BDM elements:

	$\|\|p - p_h\|\|$		$\|\|\mathbf{u} - \mathbf{u}_h\|\|$		$\|\|\nabla \cdot (\mathbf{u} - \mathbf{u}_h)\|\|$	
n	error	order	error	order	error	order
RT_0 on square meshes						
16	4.806e-01	0.91	8.579e-01	0.98	6.3e-01	0.10
36	3.244e-01	0.97	5.735e-01	0.99	4.5e-01	0.82
64	2.443e-01	0.98	4.305e-01	1.00	3.4e-01	0.92
144	1.634e-01	0.99	2.872e-01	1.00	2.3e-01	0.96
256	1.227e-01	1.00	2.154e-01	1.00	1.8e-01	0.98
RT_0 on trapezoidal meshes						
16	4.928e-01	0.93	9.175e-01	0.95	6.6e-01	0.10
36	3.327e-01	0.97	6.181e-01	0.97	4.9e-01	0.75
64	2.506e-01	0.98	4.657e-01	0.98	3.9e-01	0.77
144	1.676e-01	0.99	3.117e-01	0.99	3.0e-01	0.69
256	1.259e-01	1.00	2.341e-01	0.99	2.5e-01	0.56

Reason and fix

Piola transformation $\mathcal{P}_E$ maps a vector $\hat{\mathbf{v}} : \hat{E} \to \mathbf{R}^2$ by

$$\mathbf{v}(\mathbf{x}) = \mathcal{P}_E(\hat{\mathbf{v}})(\mathbf{x}) = \frac{1}{J_E} D\mathbf{F}_E \hat{\mathbf{v}}(\mathbf{x})$$

where $\mathbf{x} = \mathbf{F}_E(\hat{\mathbf{x}})$ and $\mathbf{F}_E$ bilinear mapping from reference to real.

- Arnold-Boffi-Falk [2005]: fix by increasing polynomial degree.
- Arbogast-Correa [2015]: directly define space on real element.
- Arbogast-Correa-Tao [coming soon...]: hexahedra.

Numerical Results & Todo

n	$\|\|p - p_h\|\|$ error	order	$\|\|\mathbf{u} - \mathbf{u}_h\|\|$ error	order	$\|\|\nabla \cdot (\mathbf{u} - \mathbf{u}_h)\|\|$ error	order
RT_0 on trapezoidal meshes						
16	4.928e-01	0.93	9.175e-01	0.95	6.6e-01	0.10
36	3.327e-01	0.97	6.181e-01	0.97	4.9e-01	0.75
64	2.506e-01	0.98	4.657e-01	0.98	3.9e-01	0.77
144	1.676e-01	0.99	3.117e-01	0.99	3.0e-01	0.69
256	1.259e-01	1.00	2.341e-01	0.99	2.5e-01	0.56
AC_0 on trapezoidal meshes (FE_ACFull)						
16	4.906e-01	0.92	9.180e-01	0.95	6.2e-01	0.19
36	3.309e-01	0.97	6.188e-01	0.97	4.5e-01	0.78
64	2.492e-01	0.99	4.663e-01	0.98	3.5e-01	0.90
144	1.667e-01	0.99	3.122e-01	0.99	2.4e-01	0.95
256	1.251e-01	1.00	2.345e-01	0.99	1.8e-01	0.98

Todo:

- Documentation.
- Parallel code: partition for hybrid style methods.
- Conforming basis.