Background

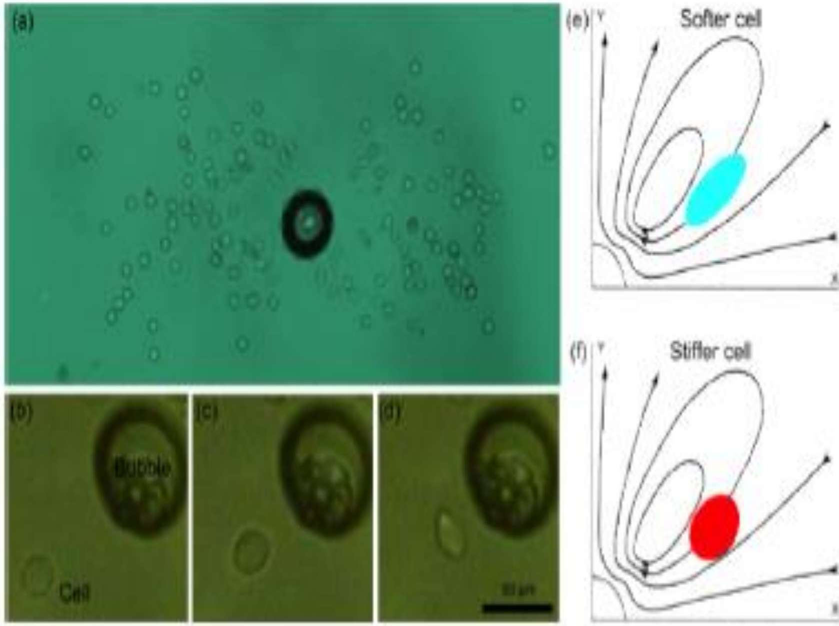
- A bubble generated inside microfluidic channel is actuated via acoustic waves, which set up a streaming flow around the bubble.
- Red blood cells introduced in the fluid will deform differently near the bubble due to different mechanical properties. (cancer cells softer, healthy cells stiffer)

AIM:

- To simulate the motion of a red blood cell (compressible object) inside the streaming flow.

Remarks:

- Wide separation of length and time scales (MHz frequencies in microscale devices).
- Navier-Stokes equations are linearized using perturbation approach, to give two set of Stokes equations.



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Immersed FEM (IFEM) for immersed solids

Balance of mass

$$\dot{\rho} + \rho \operatorname{div} u = 0, \quad x \in \Omega \setminus (\partial\Omega \cup \partial B_t),$$

Balance of momentum

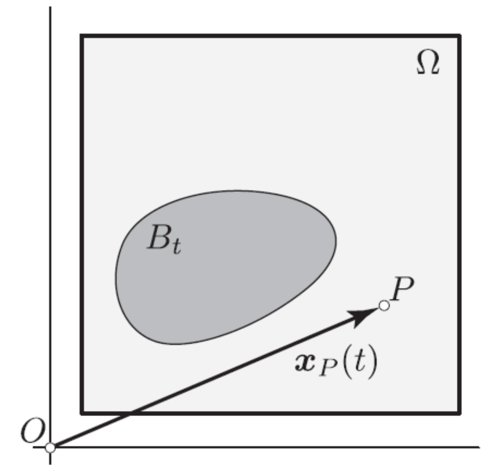
$$\operatorname{div} \mathsf{T} + \rho b = \rho \dot{u}, \quad x \in \Omega \setminus (\partial\Omega \cup \partial B_t),$$

Immersed body velocity

$$\dot{w}(s,t) = u(x,t)\big|_{x=\zeta(s,t)}$$

Boundary condition

$$u(\breve{x}^{+},t) = u(\breve{x}^{-},t) \quad \text{and} \quad \mathsf{T}(\breve{x}^{+},t)n = \mathsf{T}(\breve{x}^{-},t)n, \quad \breve{x} \in \partial$$



- Independent discretization for fluid and solid domain.
- Support of equations of motion for fluid is extended over both fluid and solid domains.
- “Body forces” terms in fluid’s equations of motion for FSI.

Balance of momentum

$$\int_{\Omega} \rho(\dot{u} - b) \cdot v \, dv + \int_{\Omega} \hat{\mathsf{T}}_{\mathrm{f}} \cdot \nabla_x v \, dv + \int_{B_t} (\hat{\mathsf{T}}_{\mathrm{s}} - \hat{\mathsf{T}}_{\mathrm{f}}) \cdot \nabla_x v \, dv - \int_{\partial\Omega_N} \boldsymbol{\tau}_g \cdot v \, da = 0 \quad \forall v \in \mathscr{V}_0$$

Balance of mass

$$\int_{\Omega} q \operatorname{div} u \, dv = 0 \quad \forall q \in \mathscr{Q}.$$

Immersed body velocity

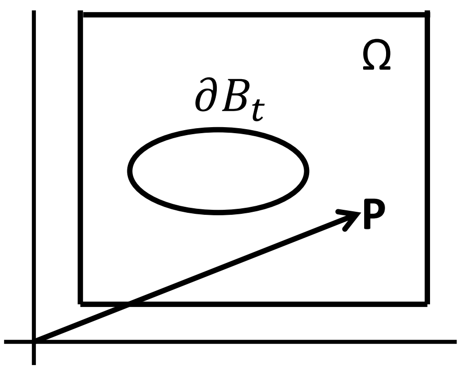
$$\Phi_B \int_B \left[\dot{w}(s,t) - u(x,t)\big|_{x=\zeta(s,t)} \right] \cdot y(s) \, dV = 0 \quad \forall y \in \mathscr{Y},$$

IFEM for codimension 1 objects

Balance of mass $\dot{\rho} + \rho \operatorname{div} u = 0, \quad x \in \Omega \setminus (\partial\Omega \cup \partial B_t),$

Balance of momentum $\operatorname{div} \mathsf{T} + \rho b = \rho \dot{u}, \quad x \in \Omega \setminus (\partial\Omega \cup \partial B_t),$

Immersed body velocity $\dot{w}(s,t) = u(x,t)\big|_{x=\zeta(s,t)}$



Boundary condition $u(\check{x}^+, t) = u(\check{x}^-, t)$ and $[\![T]\!]n = div(\sigma_s^m) \quad \check{x} \in \partial B_t,$

AIMS:

- Immersed FEM for co-dimension 1 object.
- Automatic differentiation using SACADO.
- Use SUNDIALS (SUite of Nonlinear and DIfferential/ALgebraic equation Solvers) for time-dependent solvers.