3D BDM_k Elements in deal.ii

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The Problem - Enable 3D BDM_k Basis Elements

1. BDM_k elements are used as approximation elements for $H^{div} = \{u \in L^2(\Omega) | \nabla \cdot u \in L^2(\Omega)\}$. This space appears in mixed finite element fluid problems such as Darcy's equation.
2. Elements are designed to enforce the Lagrangian property of the normal component at the nodal support points.
3. Currently, deal.ii has 2D implementations for BDM_k elements and 3D implementations for Raviart-Thomas (another H_{div} approximating element) but not 3D BDM_k.

Results

Much of the code for $3D$ elements is in place. By removing some existing asserts and modifying a line of code one can achieve the expected convergence results for 3D BDM_k elements when $k = 1$. The following are convergence rates for BDM_1 elements in 3D

cells	dofs	$\|u\|_{L^2}$		$\|p\|_{L^2}$	
8	304	1.903e+00	-	4.292e-01	-
64	2240	4.432e-01	2.10	3.013e-01	0.51
512	17152	1.395e-01	1.67	1.610e-01	0.90
4096	134144	4.107e-02	1.76	8.190e-02	0.98
32768	1060864	1.059e-02	1.96	4.112e-02	0.99

Theory suggests quadratic convergence in the velocity and linear convergence in the pressure.

Open Problems and Questions

- Implement for general k. I'm still not entirly sure of the problem; however, it appears related to how the nodal support points along the quadrilateral faces are selected.
- The method used to building these elements is rather involved. I'd like to improve the documentation in parts of the code to make it easier for new users to understand the process.