APPLICATION OF DEAL.II IN ASPECT: 3D COMPRESSIBLE MELT TRANSPORT WITH MESH ADAPTIVITY

### Juliane Dannberg<sup>1</sup>, Timo Heister<sup>2</sup>

<sup>1</sup>Texas A&M University, <sup>2</sup>Clemson University





### adaptive mesh



## **MANTLE CONVECTION**



#### From Hong Kong Geological Survey

### **MANTLE PLUMES**



From Griffiths and Campbell, 1990

Modified from Putirka et al., 2011

## MANTLE CONVECTION: MELTING



#### From Hong Kong Geological Survey

# **MELT MIGRATION**

Subduction zones



2

Mid-ocean ridges

#### Wilson et al, 2014

- Studies only in 2D / simplified
- Mantle convection and melt migration studied separately

Keller et al, 2013

Eq. (77)

2) Eq. (78)

3

Fractures/channels/diapirs

d119r2

d120r2

d122r2

DIAPIRS

1

CHANNELS

m

**C**: FRACTURES

### VARIABLES



- density
  - shear viscosity
- $\begin{array}{c} \xi^* & \text{compaction} \\ & \text{viscosity} \end{array}$
- $egin{array}{c} K_D & {\rm Darcy} \\ & {\rm coefficient} \end{array}$

# **EQUATIONS**

$$\frac{\partial}{\partial t} \left[ \rho_f \phi \right] + \nabla \cdot \left[ \rho_f \phi \mathbf{u}_f \right] = \Gamma$$
$$\frac{\partial}{\partial t} \left[ \rho_s (1 - \phi) \right] + \nabla \cdot \left[ \rho_s (1 - \phi) \mathbf{u}_s \right] = -\Gamma$$
$$\phi \left( \mathbf{u}_f - \mathbf{u}_s \right) = -K_D \left( \nabla p_f - \rho_f \mathbf{g} \right)$$
$$-\nabla \cdot \left[ 2\eta^* \left( \varepsilon(\mathbf{u}_s) - \frac{1}{3} (\nabla \cdot \mathbf{u}_s) \mathbf{1} \right) + \xi^* (\nabla \cdot \mathbf{u}_s) \mathbf{1} \right] + \nabla p_f = \bar{\rho} \mathbf{g}$$
$$\mathsf{h}_{\mathsf{h}} \xi = \mathsf{f}(\Phi)$$
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## **EQUATIONS**

$$\begin{aligned} \frac{\partial}{\partial t} \left[ \rho_f \phi \right] + \nabla \cdot \left[ \rho_f \phi \mathbf{u}_f \right] &= \Gamma \\ \frac{\partial}{\partial t} \left[ \rho_s (1 - \phi) \right] + \nabla \cdot \left[ \rho_s (1 - \phi) \mathbf{u}_s \right] &= -\Gamma \\ \phi \left( \mathbf{u}_f - \mathbf{u}_s \right) &= -K_D \left( \nabla p_f - \rho_f \mathbf{g} \right) \\ -\nabla \cdot \left[ 2\eta^* \left( \varepsilon(\mathbf{u}_s) - \frac{1}{3} (\nabla \cdot \mathbf{u}_s) \mathbf{1} \right) + \xi^* (\nabla \cdot \mathbf{u}_s) \mathbf{1} \right] + \nabla p_f &= \bar{\rho} \mathbf{g} \end{aligned}$$

Introduce compaction pressure (Keller at al, 2013):

$$p_c = (1 - \phi)(p_s - p_f)$$

## **EQUATIONS IN ASPECT**

$$-\nabla \cdot \left[2\eta^* \left(\varepsilon(\mathbf{u}_s) - \frac{1}{3}(\nabla \cdot \mathbf{u}_s)\mathbf{1}\right)\right] + \nabla p_f \qquad = \bar{\rho}\mathbf{g}$$
$$\nabla \cdot \mathbf{u}_s \qquad =$$

$$-\left( \mathbf{u}_{s}\cdot\mathbf{g}
ight)$$
  $\kappa_{s}
ho_{s}$ 

$$\frac{\partial \phi}{\partial t} + \mathbf{u}_s \cdot \nabla \phi = \frac{\Gamma}{\rho_s}$$

## **EQUATIONS IN ASPECT**

 $\Omega /$ 

$$-\nabla \cdot \left[ 2\eta^* \left( \varepsilon(\mathbf{u}_s) - \frac{1}{3} (\nabla \cdot \mathbf{u}_s) \mathbf{1} \right) \right] + \nabla p_f + \nabla p_c = \bar{\rho} \mathbf{g}$$

$$\nabla \cdot \mathbf{u}_s - \nabla \cdot K_D \nabla p_f - K_D \kappa_f \rho_f \nabla p_f \cdot \mathbf{g} = -\nabla \cdot (K_D \rho_f \mathbf{g})$$

$$+ \Gamma \left( \frac{1}{\rho_f} - \frac{1}{\rho_s} \right)$$

$$- (\mathbf{u}_s \cdot \mathbf{g}) \left[ \phi \kappa_f \rho_f + (1 - \phi) \kappa_s \rho_s \right]$$

$$- K_D \kappa_f \rho_f^2 \mathbf{g} \cdot \mathbf{g}$$

$$\nabla \cdot \mathbf{u}_s + \frac{p_c}{\epsilon_s} = 0$$

$$\frac{\partial \phi}{\partial t} + \mathbf{u}_s \cdot \nabla \phi = \frac{1}{\rho_s} + (1 - \phi) (\nabla \cdot \mathbf{u}_s + \kappa_s \rho_s \mathbf{g} \cdot \mathbf{u}_s)$$

## **CHALLENGES**

- Time and length scales of melt migration are vastly different from mantle convection
- Highly non-linear and spatially variable material properties
  - Difficult to study in 3D
  - Adaptive mesh refinement
- Link melt generation to processes in the deeper mantle (komatiites or other melts originating in greater depths)
  - Compressibility

## **SOFTWARE: ASPECT**

### **Advanced mathematical techniques:**

- Higher order time stepping schemes (BDF2)
- Higher order finite elements
- Fully adaptive, dynamically changing 3d meshes
- Nonlinear solvers
- Parallelization using MPI, threads, and tasks

### **Community code:**

- Modular
- Extensive documentation
- Extensive and frequent testing



## MODULARITY

**Aspect is very modular:** It is extended by a number of isolated "plugin" sub-systems:



## **DEAL.II FEATURES**



### **CONVERGENCE WITH ADAPTIVE MESH**



### **CONVERGENCE WITH ADAPTIVE MESH**



### SHEAR BANDS





### **SHEAR BANDS: ANGLE**



### **SHEAR BANDS IN 3D**



Total wallclock time: 1.7 hours 130 time steps; t=3.125e-08 years Number of degrees of freedom: 44,855,815

## MANTLE PLUME WITH MELT MIGRATION



Wallclock time: 7 days 14200 time steps; t=152,516 years Degrees of freedom: 6,243,260

## MANTLE PLUME WITH MELT



### MANTLE PLUME WITH MELT



## **3D PLUME: WORK IN PROGRESS**



### APPLICATIONS



# CONCLUSIONS

- Open source code for coupled magma / mantle dynamics
- Our Search Stress St
- Accurate
- Fast, scalable
- Well tested, well documented
- Oesigned to be easily extended
- Has been successfully applied to several application cases
  - On different scales
  - In three dimensions
  - > With compressibility of the individual phases

## **ASPECT TEAM**

